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INELASTIC DEUTERON SCATTERING IN THE LEAD REGION

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Synopsis

Elastic and inelastic scattering of 13.1 MeV deuterons from the stable isotopes of Bi, Pb and Tl have been studied. The scattered deuterons were observed at 120° , 125° , and 150° with a high resolution magnetic spectrograph. Angular distributions of the elastically scattered deuterons were measured and fitted with a five-parameter optical model.

A number of multiplets in the odd-mass isotopes expected on the basis of a particle-vibrational coupling model were located. Of these, the multiplets corresponding to the strong octupole states near 2.6 MeV were found in all four odd-mass nuclei and an accurate comparison of the relative population strengths was made. A large decrease in the cross section to the octupole states was observed in going from ^{208}Pb to ^{203}Tl .

1. Introduction

The nuclei in the region of ${}^{208}_{82}\text{Pb}_{126}$ have been the subject of a large number of experimental¹⁻¹³⁾ and theoretical studies¹⁴⁻¹⁷⁾ in the last few years. Of particular interest have been the low-lying levels in the nuclei which are one nucleon removed from the doubly-closed core and whose populations are found to be rather selectively dependent on the reaction used. Among these levels multiplets of states are expected on the particle-vibration coupling model¹⁸⁾ due to the coupling of the rather pure one-particle configurations to the collective vibrations of the core, and inelastic scattering studies which are particularly suited for the population of such states have succeeded in locating a number of them. The multiplets based on the strongly collective octupole state at 2.615 MeV are found to be especially strongly populated in these reactions.

In addition to the nuclei which are one nucleon removed from the doubly-closed ${}^{208}\text{Pb}$ core, those nuclei which are one nucleon removed from the closed proton-shell ${}^{204}\text{Pb}$ and ${}^{206}\text{Pb}$ cores are expected to exhibit similar multiplets. However, whereas the first level is at 2.615 MeV in ${}^{208}\text{Pb}$, a number of collective levels are known in ${}^{204}\text{Pb}$ and ${}^{206}\text{Pb}$ below this energy, and the increase in level density can be expected to produce an appreciable mixing of the various configurations.

According to the particle-vibration¹⁸⁾ coupling model, the multiplet of states obtained by coupling a one-particle configuration (spin J_p) to a 2^λ -pole oscillation of the core should be populated by transitions of multipolarity λ in the inelastic scattering whose total intensity is equal to the intensity of the corresponding state (spin λ) in the core nucleus. Furthermore, this model predicts that the inelastic scattering cross section, $d\sigma$, to each member of the multiplet (spin J) is given by

$$d\sigma(J) = d\sigma(c) \frac{2J+1}{(2J_p+1)(2\lambda+1)},$$

where $d\sigma(c)$ is the cross section to the state in the core nucleus. The "center-of-mass" energy should correspond to that of the core state. Mixing between

members of the multiplets and other states, however, is expected to exist and to cause energy shifts of the individual states and deviations from the cross section rules.

We present here results of the scattering of 13 MeV deuterons on the stable isotopes of Bi, Pb and Tl. These results, in particular for the multiplets associated with the strongly collective octupole states near 2.6 MeV, will be discussed within the scope of the particle-vibration coupling model and compared with theoretical calculations and previous experimental results where possible.

2. Experimental Procedure

The 13 MeV deuteron beam used in these experiments was obtained from the tandem accelerator at the Niels Bohr Institute. Metallic targets with thicknesses of 30 to 150 $\mu\text{g}/\text{cm}^2$ were prepared by vacuum evaporation onto $\sim 40 \mu\text{g}/\text{cm}^2$ carbon backings. The isotopic purities are listed in Table 1. During the bombardments, the targets were rotated in the plane of the target in order to reduce the deterioration found to be caused by the beam on these low-melting point metals. Particular care was taken to achieve a uniform target thickness over the enlarged area of bombardment.

The scattered beam was analyzed in a particle spectrograph whose operation has already been described¹⁹. Detection was made with 25 μ Ilford type K2 nuclear emulsion plates, which were covered with 27 mg/cm^2 of aluminium absorber. The deuteron tracks could be distinguished from the triton tracks in the few cases where the latter interfered, and were counted in 0.125 or 0.25 mm strips by means of a microscope. The scattered particles were observed at 120°, 125°, and 150°, and light element impurities were distinguished by their kinematic shifts.

The resolution obtained in these experiments was from 3 to 10 keV FWHM. The main contributions to this resolution were found to be due to straggling in the target and beam spot size. In the optimum cases, these two factors contributed about 2.0 keV (at 30 $\mu\text{g}/\text{cm}^2$) and 1.8 keV (beam spot size $0.15 \times 3 \text{ mm}^2$), respectively.

Contributions from beam energy instability over bombardment periods of ~ 15 hours appear to be less than 1.5 keV in these cases. No aluminium absorbers in front of the photographic plates were used for the highest resolution exposures and the counting of tracks was done in 0.125 mm strips only over the central portion of the emulsion. Particular care was used in choosing wrinkle-free carbon backings and the bombardments were done with the targets in a reflective geometry.

TABLE 1. Isotopic Purities

Mass Number	^{203}Tl	^{205}Tl	^{204}Pb	^{206}Pb	^{207}Pb	^{208}Pb	^{209}Bi
203	92.26	1.21					
204			99.7	< 0.01	< 0.1	< 0.05	
205	7.74	98.79					
206			0.3	99.8	2.44	0.19	
207				0.2	92.93	0.52	
208				< 0.03	4.63	99.3	
209							100

The determination of absolute cross sections was made by normalizing all transitions to the elastic peaks whose cross sections were determined from elastic angular distribution measurements. The normalization procedure used is the same as has been earlier described^{20,21}).

The experimental arrangement used in the elastic angular distribution study has been described in detail elsewhere²²). After analysis in the particle spectrograph the scattered deuterons were detected in a pair of 2 mm thick Si(Li) detectors. No problems with light element impurities in the target were encountered down to a laboratory angle of 15° .

3. Results and Discussion

The results of elastic and inelastic deuteron scattering on the stable isotopes of Bi, Pb, and Tl are presented here. Typical spectra of the deuterons scattered from these nuclei are shown in Figs. 2–10. The level energies obtained as the average of the determinations at different angles are listed in Tables 4–10, which also contain the measured differential cross sections. The individual nuclei are discussed below together with the results of the elastic scattering measurement which are shown in Fig. 1.

3.1 Elastic Scattering Distributions

Angular distributions of the elastically scattered deuterons from the seven nuclei studied were measured from 15° to 150° . Normalization of the cross sections was achieved on the assumption that the elastic scattering at 15° is pure Rutherford. This assumption seems justified in view of the optical

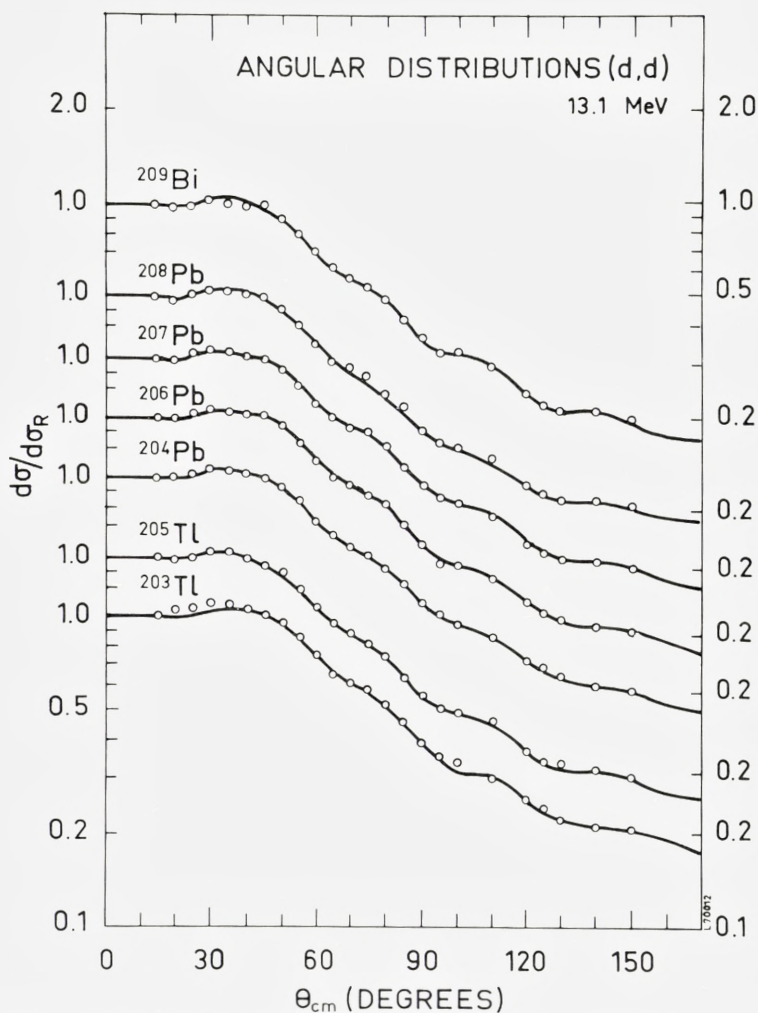


Fig. 1. Angular distributions for 13 MeV deuterons elastically scattered from nuclei in the Pb region. The curves are the best five-parameter fits.

model calculations, which find the differential cross sections to agree with the Rutherford formula at 15° within 2%. The results of the measurements are listed in Table 2. Statistical errors were kept below 2% at all angles, and allowing for $\sim 0.1^\circ$ uncertainty in setting the scattering angle and for the small error in the normalization procedure, the accuracy of the absolute cross sections is estimated to be better than 4%.

TABLE 2. Elastic Scattering Cross Sections for 13 MeV Deuterons

θ_{lab} degrees	^{209}Bi		^{208}Pb		^{207}Pb		^{206}Pb		^{204}Pb		^{203}Tl		^{203}Tl	
	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)	$\Delta\theta$	$\left(\frac{d\sigma}{d\Omega}\right)$ (mb/sr)
15	0.14	1.76 (5)	0.14	1.72 (5)	0.14	1.72 (5)	0.14	1.72 (5)	0.15	1.72 (5)	0.14	1.68 (5)	0.15	1.67 (5)
20	0.19	5.49 (4)	0.19	5.37 (4)	0.19	5.41 (4)	0.19	5.50 (4)	0.19	5.53 (4)	0.19	5.23 (4)	0.19	5.60 (4)
25	0.23	2.32 (4)	0.23	2.33 (4)	0.23	2.33 (4)	0.24	2.33 (4)	0.24	2.34 (4)	0.24	2.20 (4)	0.24	2.37 (4)
30	0.28	1.18 (4)	0.28	1.18 (4)	0.28	1.18 (4)	0.28	1.19 (4)	0.28	1.20 (4)	0.28	1.15 (4)	0.28	1.21 (4)
35	0.32	6.30 (3)	0.32	6.39 (3)	0.32	6.37 (3)	0.32	6.38 (3)	0.33	6.44 (3)	0.32	6.24 (3)	0.33	6.47 (3)
40	0.35	3.68 (3)	0.35	3.69 (3)	0.35	3.67 (3)	0.36	3.78 (3)	0.37	3.73 (3)	0.36	3.56 (3)	0.37	3.73 (3)
45	0.39	2.41 (3)	0.39	2.33 (3)	0.39	2.27 (3)	0.40	2.37 (3)	0.40	2.29 (3)	0.40	2.15 (3)	0.40	2.30 (3)
50	0.42	1.45 (3)	0.42	1.42 (3)	0.42	1.42 (3)	0.43	1.49 (3)	0.44	1.45 (3)	0.43	1.39 (3)	0.44	1.46 (3)
55	0.45	9.06 (2)	0.45	8.83 (2)	0.45	8.88 (2)	0.46	9.19 (2)	0.47	9.32 (2)	0.46	8.42 (2)	0.47	9.23 (2)
60	0.49	5.80 (2)	0.49	5.53 (2)	0.49	5.65 (2)	0.49	5.82 (2)	0.49	5.87 (2)	0.49	5.46 (2)	0.49	5.83 (2)
65	0.50	3.93 (2)	0.50	3.67 (2)	0.50	3.85 (2)	0.51	3.92 (2)	0.52	3.96 (2)	0.51	3.65 (2)	0.52	3.86 (2)
70	0.52	2.81 (2)	0.52	2.79 (2)	0.52	2.71 (2)	0.53	2.82 (2)	0.53	2.77 (2)	0.53	2.60 (2)	0.53	2.81 (2)
75	0.53	2.05 (2)	0.53	2.02 (2)	0.53	2.10 (2)	0.54	2.08 (2)	0.55	2.07 (2)	0.54	1.89 (2)	0.55	2.10 (2)
80	0.54	1.50 (2)	0.54	1.43 (2)	0.54	1.51 (2)	0.55	1.54 (2)	0.56	1.49 (2)	0.55	1.37 (2)	0.56	1.50 (2)
85	0.55	1.06 (2)	0.55	1.04 (2)	0.55	1.07 (2)	0.56	1.10 (2)	0.57	1.10 (2)	0.56	0.99 (1)	0.57	1.10 (2)
90	0.55	7.74 (1)	0.55	7.44 (1)	0.55	7.73 (1)	0.56	7.97 (1)	0.57	8.00 (1)	0.56	7.07 (1)	0.57	7.84 (1)
95	0.55	5.89 (1)	0.55	5.79 (1)	0.55	5.95 (1)	0.56	5.76 (1)	0.57	6.20 (1)	0.56	5.49 (1)	0.57	6.03 (1)
100	0.54	5.12 (1)	0.54	4.75 (1)	0.54	4.91 (1)	0.55	5.04 (1)	0.56	4.97 (1)	0.55	4.60 (1)	0.56	4.94 (1)
110	0.52	3.48 (1)	0.52	2.96 (1)	0.52	3.45 (1)	0.53	3.45 (1)	0.53	3.48 (1)	0.53	3.28 (1)	0.53	3.33 (1)
120	0.49	2.31 (1)	0.49	2.22 (1)	0.49	2.24 (1)	0.49	2.31 (1)	0.49	2.33 (1)	0.49	2.10 (1)	0.49	2.30 (1)
130	0.45	1.90 (1)	0.45	1.92 (1)	0.45	1.90 (1)	0.48	1.94 (1)	0.47	1.99 (1)	0.46	1.77 (1)	0.47	1.95 (1)
135	0.42	1.70 (1)	0.42	1.68 (1)	0.42	1.66 (1)	0.43	1.70 (1)	0.44	1.71 (1)	0.43	1.61 (1)	0.44	1.65 (1)
140	0.35	1.45 (1)	0.35	1.45 (1)	0.35	1.41 (1)	0.36	1.42 (1)	0.37	1.40 (1)	0.36	1.33 (1)	0.37	1.37 (1)
150	0.28	1.23 (1)	0.28	1.24 (1)	0.28	1.20 (1)	0.28	1.21 (1)	0.28	1.23 (1)	0.28	1.12 (1)	0.28	1.22 (1)

The number in parentheses after each cross section is the power of 10 by which the number should be multiplied to obtain the cross section in mb/sr. $\Delta\theta$ is the angle to be added to θ_{lab} to obtain the C. M. scattering angle.

TABLE 3. Five-Parameter Optical-Model Fits to the (d, d) Reaction at 13.0 MeV

Nuclide	V (MeV)	a (fm)	W (MeV)	a' (fm)	r'_0 (fm)	χ^2
^{209}Bi	98.18	0.998	11.34	0.661	1.524	1.0
^{208}Pb	97.60	1.026	17.72	0.584	1.480	2.2
^{207}Pb	100.10	0.923	12.34	0.655	1.442	0.3
^{206}Pb	100.35	0.929	15.06	0.593	1.415	0.8
^{204}Pb	98.81	0.954	13.50	0.681	1.450	0.2
^{205}Tl	99.10	0.983	12.91	0.652	1.492	0.5
^{203}Tl	100.26	0.938	16.04	0.604	1.412	1.1

$r_0 = 1.15$ fm, $r_c = 1.25$ fm.

The results have been analyzed in terms of the optical model with the parameters of best fit being obtained by using the optical-model search code JIB3 of F. C. PEREY. Details of the code and of the five-parameter search procedure have been previously published²³⁾. Figure 1 shows a comparison of the experimental results with the theoretical angular distributions calculated from the five-parameter fits. The parameters which are listed in Table 3 show very little change over the narrow mass range of study. The Q -value corrections required in order to make an accurate comparison of the relative vibrational strengths in the different nuclei have been calculated using the above parameters in the DWBA code JULIE. For the region near 2.6 MeV, this correction for $l = 3$ excitations is $\sim 7\%$ per 100 keV of excitation.

3.2. $^{208}\text{Pb}_{126}$, $^{206}\text{Pb}_{124}$, and $^{204}\text{Pb}_{122}$

The spectra of deuterons scattered from ^{208}Pb , ^{206}Pb , and ^{204}Pb are all dominated by a strong peak near 2.6 MeV of excitation. These levels, which have been previously identified as the first excited octupole states in these nuclei, are among the strongest octupole states observed. Previous inelastic scattering studies²⁾ report that, in addition to being at an almost constant excitation energy, these states have an essentially constant strength. The present inelastic deuteron scattering spectra, however, show an appreciable decrease in strength with decreasing neutron number.

Above the octupole state in ^{208}Pb a second strongly populated level is observed. This is the $5-$ level at 3.18 MeV. Several $5-$ levels are reported in ^{206}Pb in this energy region, but a strongly populated state of this spin and parity is not observed until 3.78 MeV. The lowest $5-$ level in ^{208}Pb is dominated by the $(g_{9/2}, p_{1/2})$ one-neutron particle-hole configuration¹⁴⁾ and a change from ^{208}Pb to ^{206}Pb , which affects mainly the $p_{1/2}$ neutrons, causes a

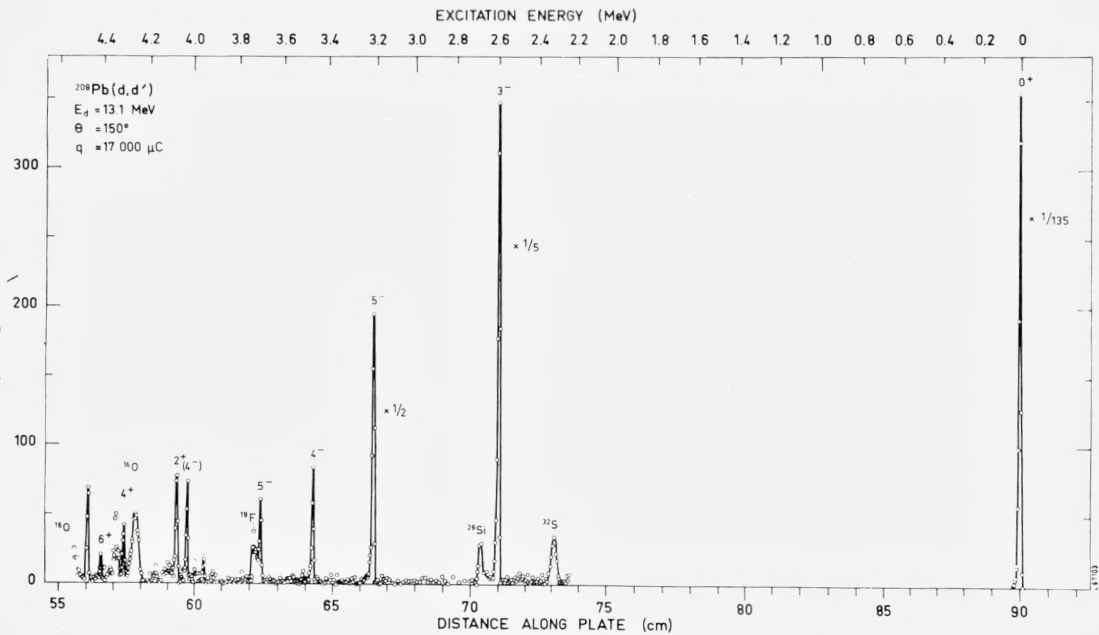


Fig. 2. Spectrum of deuterons scattered from ^{208}Pb .

TABLE 4. Levels Populated in ^{208}Pb

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega) 125^\circ$ ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega) 150^\circ$ ($\mu\text{b}/\text{sr}$)	Assignment
0	0	19200	12400	0+
2.614	2.614	350	393	3-
3.198	3.198	74	85	5-
3.475	3.476	11	16	4-
3.707	3.709	9	8	5-
3.959	3.961		4	
4.037	4.025		14	(4-)
4.083	4.070	12	16	2+
4.320	4.305	8	9	4+
4.358			11	
4.421	4.405		3	6+
4.477	4.465		13	

a) ref. 3).

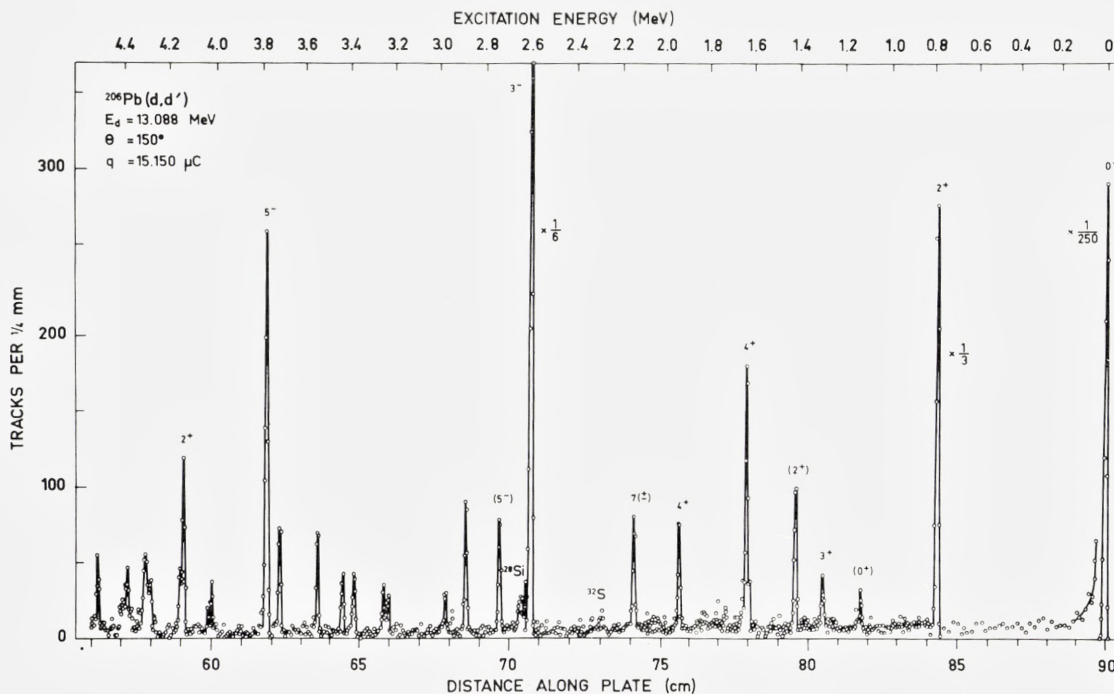


Fig. 3. Spectrum of deuterons scattered from ^{206}Pb .

large increase in the energy of this configuration. However, as previously pointed out¹⁾, it is not clear whether there exists a simple correspondence between these states in the two nuclei. In the present spectra, the 3.78 MeV state in ^{206}Pb is populated with a cross section that is only about 40 % of that to the 3.18 MeV state in ^{208}Pb . No dyotriacontapole vibration states have yet been identified in ^{204}Pb in this energy region.

In contrast to ^{208}Pb where the first excited state is found at 2.614 MeV, a number of relatively low-lying levels are observed in ^{206}Pb and ^{204}Pb . The first excited state in both these nuclei is the 2^+ level at 0.803 and 0.900 MeV, respectively. These states which are predominantly two-neutron hole and four-neutron hole levels, respectively, do contain some collective strength and are appreciably populated in inelastic scattering. Of the remaining low-lying states presently observed, only the 4^+ levels at 1.69 MeV in ^{206}Pb and 1.27 MeV in ^{204}Pb are relatively strongly populated.

It is perhaps worth noting that the cross section for the unnatural parity 4^- state at 3.48 MeV in ^{208}Pb is greater than that for the 5^- level at 3.71 MeV

TABLE 5. Levels Populated in ^{206}Pb

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega)$ 125° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 150° ($\mu\text{b}/\text{sr}$)	Assignment
0	0	19400	12100	0+
0.803	0.803	141	133	2+
1.163	1.16	4	3	(0+)
1.340	1.341	4	5	3+
1.464	1.459	16	14	(2+)
1.680	1.684	23	26	4+
1.993	1.996	9	10	4+
2.197	2.199	7	11	(7±)
2.649	2.648	291	309	3-
2.782	2.787	8	11	(5-)
2.925	2.931	10	12	(4+)
3.014	3.020	2	3	(5-)
3.256	3.267	2	3	
3.276		3	4	
3.400	3.403	4	6	5-
3.450	3.453	4	5	
3.559	3.560	7	9	(5-)
3.719	3.721	10	9	
3.774	3.776	29	31	5-

a) ref. 3).

even though, at back angles, it is predominantly populated by a (deuteron) spin flip. The inelastic proton scattering³⁾, on the other hand, seems to yield less of the 4- state compared to the 5- at 3.71 MeV.

3.3. $^{209}_{83}\text{Bi}_{126}$

The low-lying levels in ^{209}Bi have been previously investigated by a number of one-particle transfer and inelastic scattering studies. The proton transferring (α, t)²⁴⁾ and ($^3\text{He}, d$)^{4,25,26)} reactions on ^{208}Pb targets are found to appreciably populate only the single-particle states at 0.0 ($h_{9/2}$), 0.897 ($f_{7/2}$), 1.608 ($i_{13/2}$), 2.314 ($f_{5/2}$), 3.108 ($p_{3/2}$), and 3.624 MeV ($p_{1/2}$) below 4.0 MeV of excitation. In striking contrast, the inelastic deuteron²⁷⁾ and proton¹³⁾ spectra are dominated by strong groups near 2.62 and 3.18 MeV with very little population of the one-particle levels.

On the basis of the particle-vibration coupling model, the coupling of the $h_{9/2}$ ground-state proton in ^{209}Bi to the 3- core vibration should produce a septuplet of positive parity levels near 2.62 MeV with spins ranging from

TABLE 6. Levels Populated in ^{204}Pb

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega)$ 125° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 150° ($\mu\text{b}/\text{sr}$)	Assignment
0	0	19900	12300	0+
0.899	0.899	193	185	2+
1.272	1.274	38	39	4+
1.351		11	12	
1.561	1.563	7	9	(4+)
1.579		4	4	
1.663		8	10	
1.816		19	17	
1.871		3	7	
2.156		3	3	
2.180	2.186	2	3	9-
2.256	2.255	19	33	
	2.258			
2.508		2	3	
2.618		225	268	3-
2.804		4	6	
2.884		6	9	
2.896		10	12	
3.561		12	15	
3.719		2	5	
3.778		4	4	
3.799		3	3	
3.824		3	3	
3.951			11	
4.004			9	

a) ref. 42).

$J = 3/2$ to $15/2$ whose relative population cross sections should be proportional to $2J + 1$. HAFELE and WOODS¹³⁾ were able to locate six of the seven members and made the initial spin assignments for the multiplet, using this intensity rule and assuming that the strongest populated peak at 2.600 MeV (cf. Fig. 6) is a doublet containing the $11/2+$ and $13/2+$ states. Present attempts to resolve this doublet have been unable to distinguish any broadening of this peak relative to the other presumably single peaks even at a resolution of 3.5 keV, and an upper limit of 1.6 keV is placed on energy spacing of the two members. Recent Coulomb excitation experiments employing 70 MeV $^{16}\text{O}^{12)}$ and 19 MeV α -beams^{11 12)} report this spacing as 2.0 ± 1.5 keV on the basis of the energies of several highly Doppler-broadened gamma transitions.

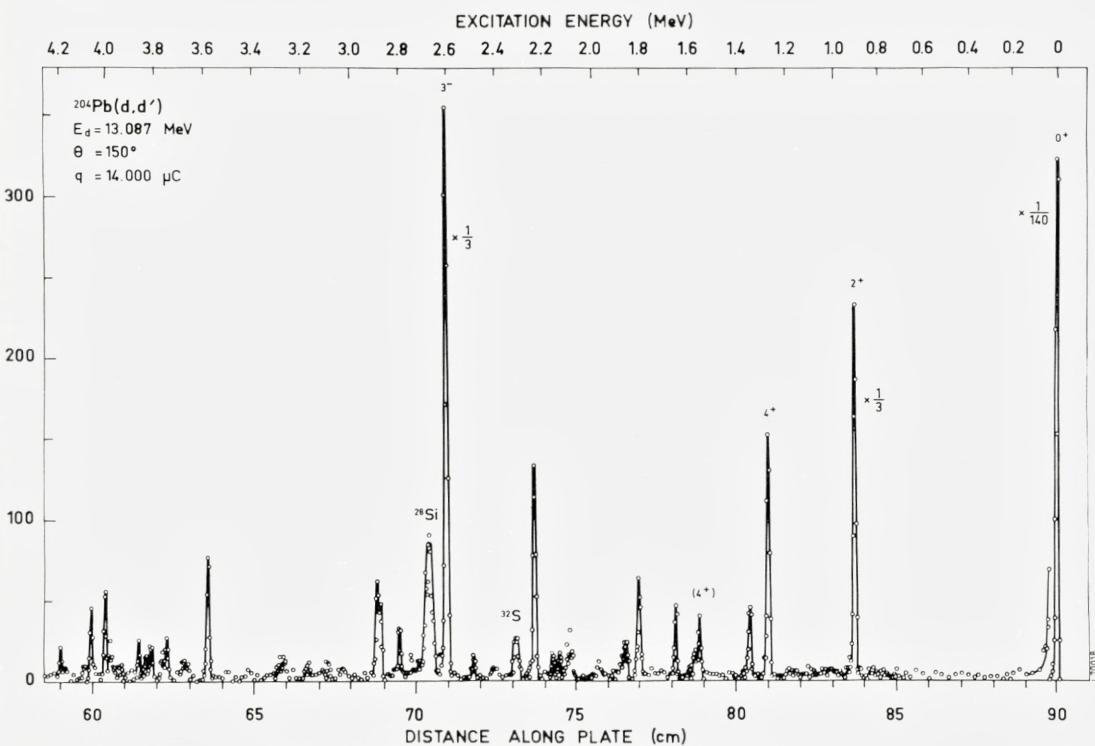


Fig. 4. Spectrum of deuterons scattered from ^{204}Pb .

The spin assignments shown in Fig. 6 are the same as those first proposed by HAFELE and WOODS¹³⁾. In addition to the excellent adherence to the $2J+1$ rule (cf. Table 14), the observed gamma decay of the Coulomb-excited septuplet^{11,12)} and the recent resonance fluorescence measurements of METZGER⁸⁾ offer strong evidence in support of this spin sequence.

A number of attempts have been made to calculate the shifts in energy of the multiplet members from the unperturbed position. The results of HAMAMOTO¹⁵⁾ using a perturbation theory approach and of BROGLIA et al.¹⁶⁾ using the Brown G-matrix method are shown in Fig. 11, together with the experimental results. Also shown are the recent results of ARITA and HORIE²⁸⁾ who have used a shell-model approach without the assumption of weak particle-vibration coupling. The large number of small and cancelling contributions due to the many possible admixtures make the calculations somewhat sensitive to the individual components, and no good reproduction of the experimentally observed level order has yet been produced. Neither of the

TABLE 7. Levels Populated in ^{209}Bi

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega)$ 120° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 125° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 150° ($\mu\text{b}/\text{sr}$)	Assignment
0	0	23100	19000	12300	9/2-
0.900	0.897	1	1	1	7/2-
1.608	1.609	14	14	13	13/2+
2.494	2.493	12	13	13	3/2+
2.565	2.563	47	45	48	9/2+
2.585	2.584	36	40	42	7/2+
2.600	2.600	119	116	121	11/2+, 13/2+
	2.602				
2.618	2.617	28	30	34	5/2+
2.744	2.741	69	65	72	15/2+
2.768		3	2	3	5/2-
2.828		4	4	5	
2.958		2	3	4	
2.988		9	8	10	(13/2+)
3.041		3	3	4	(3/2+)
3.091		3	4	4	(5/2+)
3.136		19	19	21	(11/2+, 19/2+)
3.154		14	14	16	(17/2+, 7/2+)
3.170		8	8	12	(15/2+)
3.213		6	5	6	(9/2+)
3.308		2		4	
3.379		1		1	
3.407		2		2	
3.466		4		5	

a) ref. 12).

two particle-vibration approaches is able to account for the large depression of the 3/2+ multiplet member. However, it may be noted that all three approaches do agree on the spin 15/2 state being the one of highest energy as is in fact observed, and do predict the overall energy spread of the multiplet.

It is interesting to note that, although the $f_{7/2}$ state at 0.897 MeV, which may be populated from the $h_{9/2}$ ground state by an E2 transition, is not observed in the present spectra (cf. Fig. 5), a significant population of the $i_{13/2}$ level at 1.608 MeV, which is the only low-lying positive parity particle state, is observed. As previously noted²⁷⁾, this is probably an indication of the mixing into this state of the 13/2+ member of the octupole multiplet. Such a mixing has been used by BROGLIA, DAMGAARD and MOLINARI¹⁶⁾ in calculating the $B(\text{E}3)$ value to the $i_{13/2}$ level. Using an amplitude $\varepsilon = 0.22$

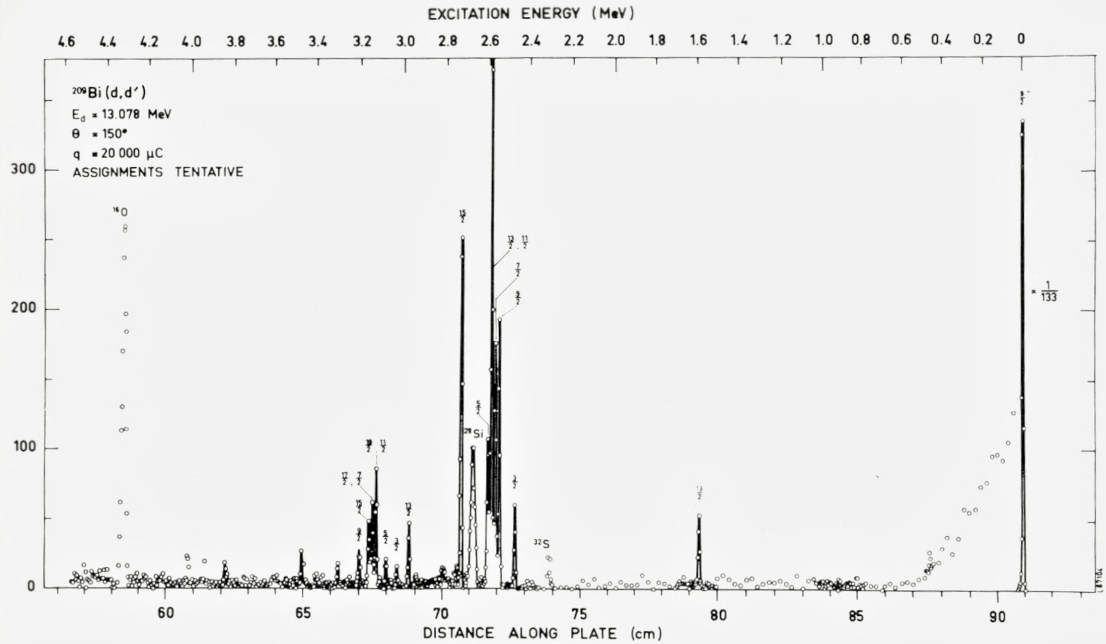


Fig. 5. Spectrum of deuterons scattered from ^{209}Bi .

of the $13/2^+$ multiplet state into the single-particle level, they calculate $B(E3, 9/2 \rightarrow 13/2) = 1.6 \times 10^4 e^2 fm^6$. This is in good agreement with the value $(1.24 \pm 0.32) \times 10^4 e^2 fm^6$ reported from Coulomb excitation¹¹⁾ and with the value $1.4 \times 10^4 e^2 fm^6$ deduced from the present results by use of the empirical relationship found to exist between $B(E\lambda)$ values and inelastic deuteron cross sections²⁹⁾. An upper limit for the mixing of these two states can be calculated by assuming that all the observed (d, d') intensity at 1.608 MeV comes from the octupole component. The percent mixing is then just the cross section for excitation of this level divided by the sum of the cross sections for excitation of the $i_{13/2}$ and $13/2^+$ multiplet states after correction for the difference in the yields due to the difference in Q -value for the two levels. Using 7/13 of the cross section to the unresolved group at 2.600 MeV, as predicted by the $2J+1$ rule, this comes out 13%, but is only an upper limit as there is some excitation of the single-particle state directly. A rough estimate of this leads to a mixing probability $\varepsilon^2 \sim 0.07$. It is also worth noting that a weak high-spin group has been observed at 2.61 MeV in both the $^{208}\text{Pb}(\alpha, t) ^{209}\text{Bi}$ ²⁴⁾ and $^{208}\text{Pb}(^3\text{He}, d) ^{209}\text{Bi}$ ^{4,25,26)} transfer reaction spectra. Such a peak would be expected to arise from the admixture of the $i_{13/2}$ state into the $13/2^+$ multiplet

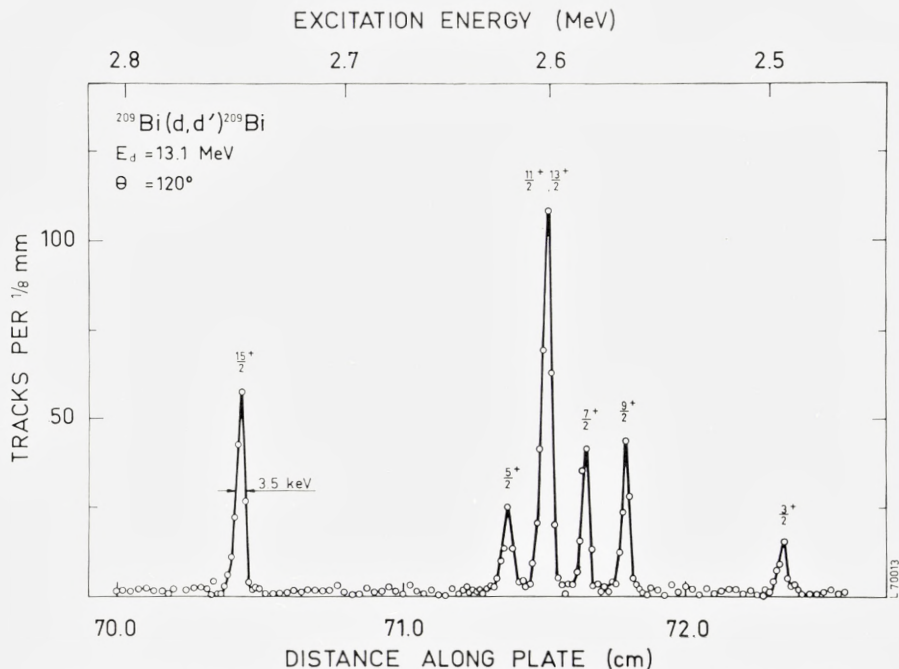


Fig. 6. Spectrum of deuterons scattered from the octupole multiplet in ^{209}Bi .

member. From its intensity relative to the strong group at 1.608 MeV, LILLEY and STEIN²⁴) were able to calculate the percent mixing as about 9% (in this case an actual number and not a limit, if, as expected, there is no core excitation). In a similar manner, ELLEGAARD and VEDELSBY⁴) extract a value of 6% from their ($^3\text{He}, d$) results.

Above the octupole multiplet, a number of weaker groups are observed in the present spectra. The weak transition at 2.828 MeV corresponds to the very strong transitions observed in $^{208}\text{Pb}(\alpha, t)$ ^{209}Bi ²⁴) and $^{208}\text{Pb}(^3\text{He}, d)$ ^{209}Bi ^{4, 25, 26}) studies and assigned to the $f_{5/2}$ proton state. The strong groups around 3.2 MeV (cf. Fig. 7) must correspond to the collective 5- level in ^{208}Pb . In ^{209}Bi , the coupling of an $h_{9/2}$ proton to the dyotriacontapole vibration should yield ten positive parity levels ranging from 1/2 to 19/2 in spin. The blocking of one $h_{9/2}$ orbital is not expected to damage the wave function¹⁴) of this state any more than that of the first 3- state, and so the cross section for excitation of the multiplet should be almost as high as that of the corresponding state in ^{208}Pb . No published calculations exist at present on the expected energy spectrum of this multiplet. However, the admixtures are expected to be smaller than in the octupole case and so the energy spread

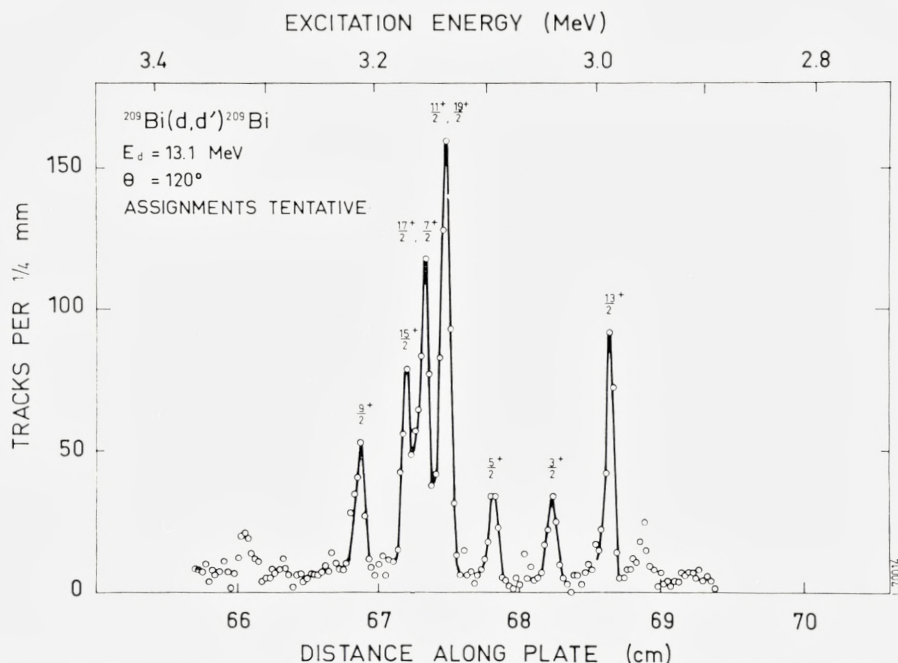


Fig. 7. Spectrum of deuterons scattered from the dyotriacontapole multiplet in ^{209}Bi . This spectrum is the sum of several exposures at 120° .

is expected to be less (about 150 keV)³⁰⁾ and the adherence to the $2J+1$ rule for population intensities should hold well.

The tentative assignments shown in Fig. 7 have been made on the basis of excitation cross sections. The agreement with prediction is shown in Table 11 for this and also a second possible choice. Of the ten peaks expected in the multiplet, the $1/2+$ member is expected to be low in intensity and has not been assigned in the spin sequence proposed. Seven peaks, whose total cross section is $(76 \pm 5)\%$ of the cross section to the corresponding $5-$ peak in ^{208}Pb , are then used to account for the remaining levels. Higher resolution spectra than those presented on this multiplet, but with somewhat decreased statistics, have resolved the peak at 3.154 MeV into two components about 4 keV in separation. Two levels are assumed also to exist at 3.136 MeV because of the large population of the group at that position. However, these two states have not yet been resolved and only an upper limit of 3.0 keV can be placed on their separation. Further experimental studies including angular distributions which might determine the states definitely belonging to the multiplet are required before a conclusive identification can be made.

TABLE 8. Levels Populated in ^{207}Pb

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega)$ 125° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 150° ($\mu\text{b}/\text{sr}$)	Assignment
0	0	19000	12000	1/2-
0.570	0.570	45	37	5/2-
0.897	0.897	54	38	3/2-
1.625	1.634	8	5	13/2+
2.333	2.332	9	14	7/2-
2.616	2.610	136	153	5/2+
2.656	2.655	169	193	7/2+
2.721	2.725	18	17	9/2+
3.188		3	4	
3.205	3.200	2	3	
3.377	3.380	8	10	
3.402	3.405	1	2	
3.415		1	1	
3.426	3.430	1	2	
3.469		2	3	
3.503	3.505	7	6	
3.578	3.575	7	7	
3.611	3.615	6	8	
3.625		5	5	
3.644	3.640	7	7	
3.898	3.890	4	5	
4.094	4.090	7	8	3/2-
4.133	4.125	8	8	5/2-

a) Refs. 3) and 7).

3.4. $^{207}_{82}\text{Pb}_{125}$

The spectrum of deuterons scattered from ^{207}Pb (cf. Fig. 8) is dominated by the two states at 2.616 MeV (5/2+) and 2.656 MeV (7/2+) produced by the coupling of the ground-state $p_{1/2}^{-1}$ neutron configuration to the octupole core vibration. These states are expected to be quite pure³¹⁾ and have so far been observed only in scattering experiments. A similar doublet of states ($J\pi = 9/2+, 11/2+$) is expected at 3.2 MeV due to a coupling of the ground state to the 5- state in ^{208}Pb . Angular distributions corresponding to $l = 5$ angular momentum transfer have been measured for inelastic proton scattering³⁾ from the states at 3.205 and 3.377 MeV. The inelastic deuteron cross section presently observed to these two levels, however, is 4 to 6 times smaller than that to the corresponding 5- state in ^{208}Pb , indicating that if these two

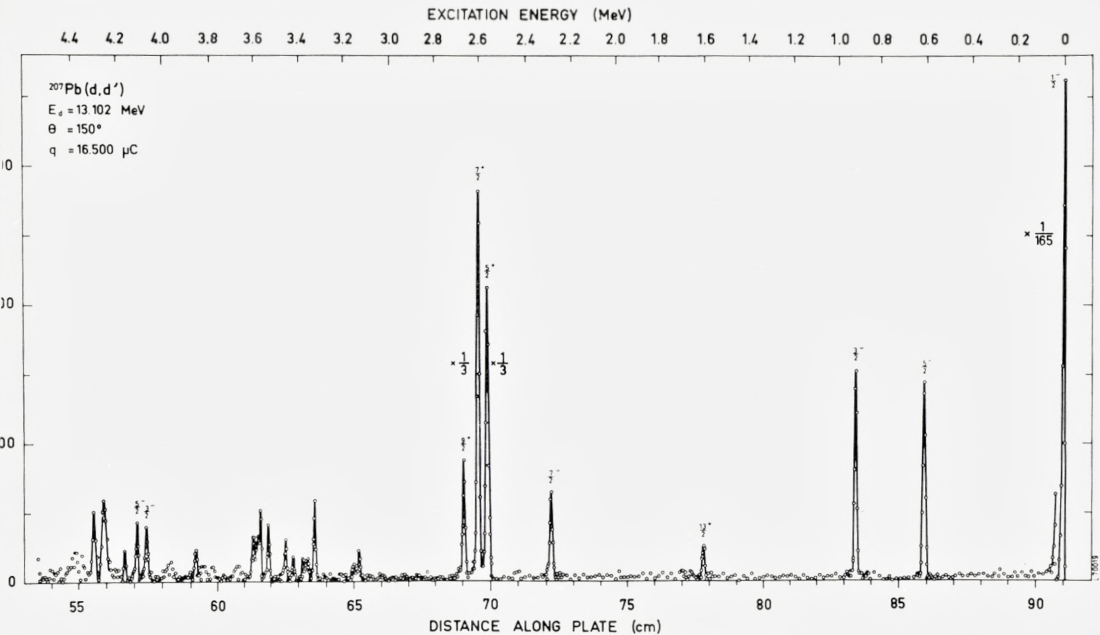


Fig. 8. Spectrum of deuterons scattered from ^{207}Pb .

states do in fact form the $5-$ doublet, then a substantial dilution of the collective strength exists. It should be noted that the one-particle $g_{9/2}$ and $i_{11/2}$ neutron configurations have been observed in this region of excitation. These configurations, which have the proper spin and parity to mix with the expected $9/2+$ and $11/2+$ doublet states, are found to be somewhat fragmented³²⁾ and may contain a substantial portion of the collective strength. The state at 2.730 MeV which contains the major portion of the $g_{9/2}$ strength^{7,9,33)} is in fact populated with a cross section which is comparable to the combined cross section observed to the 3.205 and 3.377 MeV levels.

Spin-parity assignments of $3/2-$ and $5/2-$ have been previously made³⁾ for the states at 4.094 and 4.133 MeV, respectively. This doublet, interpreted as arising from the coupling of the ground state to the $2+$ ^{208}Pb core vibration at 4.078 MeV, is populated in the present experiment with a cross section essentially equal to the cross section to the $2+$ state.

In addition to the expected collective levels, the first two excited states are rather strongly excited in the present study. The measured spectroscopic factors for the excitation of these states in neutron^{7,32)} pick-up reactions are in agreement with pure one-hole configurations, and the relatively large cross sections observed in this study are therefore surprising. It is interesting to

note that a somewhat similar situation has been found in the scattering of 20 MeV protons from ^{207}Pb by GLASHAUSSER et al.⁶⁾ There it was found that appreciable polarization of the ^{208}Pb core was required to account for the excitation of the observed hole states. It should also be noted that the ratio of the inelastic deuteron cross sections to the $B(E2)$ values which is found to be fairly constant in the rare-earth region²⁹⁾ is a factor of three larger for these two states than for other levels in the present mass region.

It appears that this effect can be ascribed to the polarization of the lead core by the neutron hole. The deuteron excites the polarization charge together with the single-neutron transition whereas only the polarization contributes to the matrix element measured by Coulomb excitation. The effective charge of the neutron hole is known to be close to unity and is mostly of isoscalar type (equal participation of protons and neutrons). On this basis one estimates a value for the above-mentioned ratio of $(3/2)^2 = 2.25$ in approximate agreement with the observed value.

3.5. $^{205}_{81}\text{Tl}_{124}$

The levels of the odd-mass Tl isotopes have been investigated recently in a number of experimental^{5,34,35,43)} and theoretical^{17,36,37)} studies. The proton pick-up studies of HINDS et al.³⁴⁾ indicate that the ground-state configurations in ^{203}Tl and ^{205}Tl have spectroscopic factors somewhat smaller than those expected for pure $3s_{1/2}^{-1}$ shell-model states. This factor together with the relatively large number of low-spin positive parity states expected below 1.5 MeV of excitation make the description of these nuclei somewhat more complex than first proposed by DE SHALIT³⁸⁾ who considered simply a coupling of the ground-state proton to the vibrations of the Hg core.

The coupling of the $s_{1/2}^{-1}$ proton configuration of the ^{205}Tl ground state to the $2+$ vibration of the ^{206}Pb core is expected to produce a low-lying doublet ($J\pi = 3/2+, 5/2+$) of levels which should be strongly populated in inelastic scattering. Two states which are known to have these spin and parity assignments are in fact strongly populated in the low excitation energy portion of the present spectra. However, it must be noted that the $d_{3/2}^{-1}$ shell-model configuration is expected to be found in these nuclei and should strongly mix with the $3/2+$ member of the doublet. The (t, α) spectra of HINDS et al.³⁴⁾ show a strong population of the $3/2+$ state at 0.203 MeV, which the authors report to contain about 60 % of the cross section predicted for a $2d_{3/2}$ proton hole. They also observe a second $3/2+$ state at 1.14 MeV and suggest that this

TABLE 9. Levels Populated in ^{205}Tl

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega)$ 120° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 125° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 150° ($\mu\text{b}/\text{sr}$)	Assignment
0	0	21000	17700	11200	1/2+
0.203	0.205	75	62	63	3/2+
0.616	0.615	148	124	116	5/2+
0.920		15	15	19	
1.136	1.14	11	10	8	3/2+
1.174		3		2	
1.336	1.34	7		6	
1.426	1.43	32	30	31	
1.479	1.48	6		5	
1.571	1.58	1		1	
1.637		3		2	
1.668		1		1	
1.768		2		2	
1.858	1.86	3		5	
2.482	2.49	31	28	35	
2.623	2.61 ^{b)}	75	76	85	5/2-
2.717	2.69 ^{b)}	96	89	107	7/2-
2.933		4			
2.974		4			
3.173		6		8	
3.213		12		12	
3.259		26	25	29	
3.411		8		9	
3.473		2		4	
3.523		4		4	
3.540		10		14	

a) ref. 34). b) ref. 39).

may be the major component of the 3/2+ doublet member. Such an order of these two 3/2+ levels is also in agreement with the calculations of COVELLO and his co-workers¹⁷⁾ who also have calculated the reduced E2-transition probabilities to the low-lying states in ^{205}Tl and predict a value of 0.15 for the ratio of the $B(E2)$ to the upper 3/2+ state to that to the lower. The corresponding ratio of cross sections presently observed is 0.16 and this factor together with the 120° to 150° cross section ratio, which is in agreement with the transition being $l = 2$, support such an assignment for the 1.137 MeV state. The strongly populated level at 0.615 MeV has been previously assigned as the major component of the 5/2+ doublet member. A weak-coupling

prediction of this state would require that it contains 60 % of the 2+ inelastic cross section; the observed value is 80 %.

A low-lying quadruplet of levels ($J\pi = 1/2+ \text{ to } 7/2+$) whose wave functions contain large components of the $2d_{3/2}^{-1}$ shell-model state coupled to the 2+ core vibration is expected¹⁷⁾ in both ²⁰³Tl and ²⁰⁵Tl. The lowest-lying member of this quadruplet is predicted to be the 7/2+ which also contains a substantial portion of the ground state coupled to the 4+ vibrational state in its calculated wave function¹⁷⁾. A level at 0.920 MeV is seen in the present spectra which has recently been reported to be populated by an $l = 4$ transition in the (p, p') studies of GLASHAUSSER et al.⁵⁾. This level, which then is restricted to spin 7/2+ or 9/2+, is populated in the present spectra with a cross section (Q -value corrected) which is 59 % of the cross section observed to the 4+ level at 1.684 MeV in ²⁰⁶Pb. The (p, p') experiment⁵⁾ reports this ratio to be 45 %. A 7/2+ assignment for this state as suggested by comparison with the calculated spectra is consistent with the proton pick-up results of HINDS et al. These authors see no population of the 0.920 MeV level. COVELLO and co-workers¹⁷⁾ calculate that the strength of the $2g_{7/2}^{-1}$ component in the low-lying 7/2+ state (predicted at 0.85 MeV) is only 1.5 %.

A state at 1.21 MeV is reported by HINDS et al.³⁴⁾, which they suggest may contain the major component of the 1/2+ member of the quadruplet. No population of this state, which would be populated directly only via the admixture of the excited level into the ground state, is observed in the present spectra.

The two remaining quadruplet members, the 3/2+ and 5/2+, have spins appropriate to admix with the strongly populated doublet formed by coupling the ground state to the 2+ core vibration and would be expected to contain some inelastic cross section. A number of yet unidentified states are seen below 1.7 MeV, which may correspond to these levels. Of these, the two most strongly populated, at 1.336 and 1.426 MeV, appear to be observed also in the (γ, γ') reaction^{35,43)} where they are directly populated from a 1/2- resonance state and hence are restricted to spins 1/2, 3/2 or 5/2. However, no conclusive assignments yet seem possible. The only other state identified below 2.0 MeV, the $h_{11/2}$ at 1.479 MeV, is only slightly populated.

The higher energy portion of the spectrum is dominated by a doublet of peaks near 2.65 MeV, which is undoubtedly the configuration produced by a coupling of the 1/2+ ground state to the octupole vibration. These two states at 2.623 and 2.717 MeV were previously observed in inelastic proton studies³⁷⁾ and were given respective spin assignments of 5/2- and 7/2- on the basis of the $2J + 1$ cross section rule. SOLF et al.³⁹⁾ suggest that the doublet should

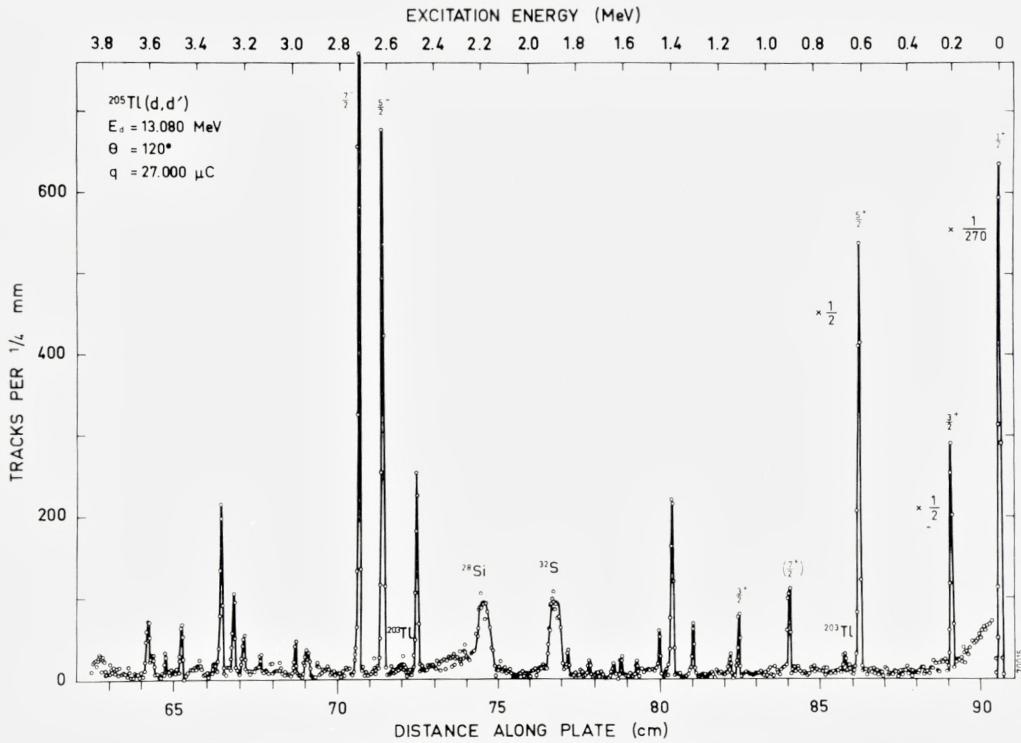


Fig. 9. Spectrum of deuterons scattered from ^{205}Tl .

contain no more than about 70 % of the ^{206}Pb cross section to the octupole state. The present ratio observed is 62 %. Three additional levels which are populated by $l = 3$ transitions have been recently⁵⁾ identified in ^{205}Tl . These states, at 2.482, 3.213, and 3.259 MeV, are, except for the above-mentioned octupole doublet, the most strongly populated states above 2.0 MeV of excitation. The relative cross sections presently observed for the five octupole transitions are in good agreement with those reported by GLASHAUSSER et al.⁵⁾ As pointed out by these authors, a quadruplet of levels ($J\pi = 3/2^-$ to $9/2^-$) is expected about 0.35 MeV above the previously identified octupole doublet due to a coupling of the $2d_{3/2}^{-1}$ state to the 3^- level. Of these four levels the $5/2^-$ and $7/2^-$ might be expected to admix with the strongly populated 2.623 and 2.717 MeV state and to then be populated in inelastic scattering. The only other $5/2^-$ or $7/2^-$ levels expected in this region of excitation, which might be populated via admixtures with these states, would be due to a coupling of the $h_{11/2}$ configuration to the 2^+ state. However, the excitation of such a configuration is not expected to be very large.

The inelastic cross section observed to the octupole doublet in ^{205}Tl is somewhat less than observed to the $3-$ levels near 2.65 MeV in both ^{204}Pb and ^{206}Pb (see Table 13) as well as in $^{204}\text{Hg}^{41)}$, indicating that there may, in fact, be some of the inelastic cross section carried to other levels. If all five octupole levels are considered, the total octupole cross section observed in ^{205}Tl is 91 % of that observed to the 2.649 MeV state in ^{206}Pb . As a final comment, it may be noted that, while the ratio of 0.75 predicted by the weak-coupling model for the cross section to the $5/2-$ state to that to the $7/2-$ state would probably be disturbed by the substantial admixtures required to explain the appreciable population observed to the additional octupole states, a value of 0.73 ± 0.03 is presently observed.

3.6. $^{203}_{81}\text{Tl}_{122}$

The low excitation energy portion of the spectrum of deuterons scattered from ^{203}Tl is dominated by strong peaks corresponding to the first two excited states. As in the case of ^{205}Tl , a low-lying $d_{3/2}^{-1}$ shell-model state is expected in this region of excitation together with the doublet of states due to a coupling of the $1/2+$ ground state to the $2+$ vibrational state at 0.900 MeV in ^{204}Pb . The first excited state in ^{203}Tl is the $3/2+$ level at 0.280 MeV. The strong population of this level in the inelastic deuteron spectrum (cf. Fig. 10) suggests that it may be the $3/2+$ member of the vibrational doublet. However, both the (t, α) results of HINDS et al.³⁴⁾ which indicate that this level contains about 65 % of the $2d_{3/2}^{-1}$ proton configuration, and the gamma-decay results of KARLSSON⁴⁰⁾, which yield a magnetic moment for the level near the Schmidt limit, indicate that the major component of the vibrational state must be found elsewhere. No higher-lying $3/2+$ state has yet been identified in this nucleus. As in ^{205}Tl , it is predicted¹⁷⁾ that the $B(E2)$ and hence inelastic cross section to this second $3/2+$ level is only about 15 % of that to the predominantly $d_{3/2}^{-1}$ state at 0.280 MeV. The states at 1.042 and 1.073 MeV both are populated with cross sections of this magnitude, and from their 120° to 150° cross section ratio are consistent with $l = 2$ momentum transfers. However, a conclusive identification of the predominantly vibrational $3/2+$ level with either of these states is not yet possible.

The only other positively identified state in ^{203}Tl is the $5/2+$ level at 0.680 MeV. As previously pointed out³⁴⁾, this state probably contains the main component of the $5/2+$ quadrupole doublet member. It is interesting to note that, within experimental uncertainty, the cross section to this state alone is equal to that to the $2+$ state in ^{204}Pb .

TABLE 10. Levels Populated in ^{203}Tl

Energy (MeV)	Previous energy ^{a)} (MeV)	$(d\sigma/d\Omega)$ 120° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 125° ($\mu\text{b}/\text{sr}$)	$(d\sigma/d\Omega)$ 150° ($\mu\text{b}/\text{sr}$)	Assignment
0	0	23300	19500	12200	1/2+
0.280	0.285	107	89	85	3/2+
0.680	0.690	195	177	175	5/2+
1.042		13	11	8	
1.073	1.07	12	9	11	
1.181		7	5	4	
1.210		17	17	23	
1.228	1.24	9	11	6	
1.262		10	8	9	
1.386		5		4	
1.446		13	13	16	
1.481	1.47	16	16	20	
1.861			12	10	
2.430		3		4	
2.483		29	33	39	5/2-
2.539		53	51	62	7/2-
2.683				16	
2.828		11		17	
2.893		27		37	
2.954		16	16	19	
3.081		18		23	
3.110		8		10	

a) ref. 34).

A number of relatively weakly populated states are found between 1.0 and 2.0 MeV of excitation and then at 2.483 and 2.539 MeV comes a strongly populated doublet of levels. The energy of this doublet and the magnitude of the cross sections observed make these immediate candidates for the octupole doublet expected in this region. On the basis of the $2J+1$ cross section rule, spin assignments of $5/2-$ for the 2.483 MeV state and $7/2-$ for the 2.539 MeV state are proposed. This rule predicts a ratio of 0.75 for the cross section of the $5/2-$ state to the $7/2-$ state. The observed ratio which is 0.60 ± 0.03 indicates that there is appreciably more mixing into the octupole doublet in ^{203}Tl than in ^{207}Pb and ^{205}Tl .

A substantial population of the states between 2.4 and 3.5 MeV is observed in the present study. As in ^{205}Tl , the quadruplet expected by coupling the $d_{3/2}^{-1}$ configuration to the $3-$ state may correspond to these states which are observed in the inelastic scattering via the admixed components. The ratio

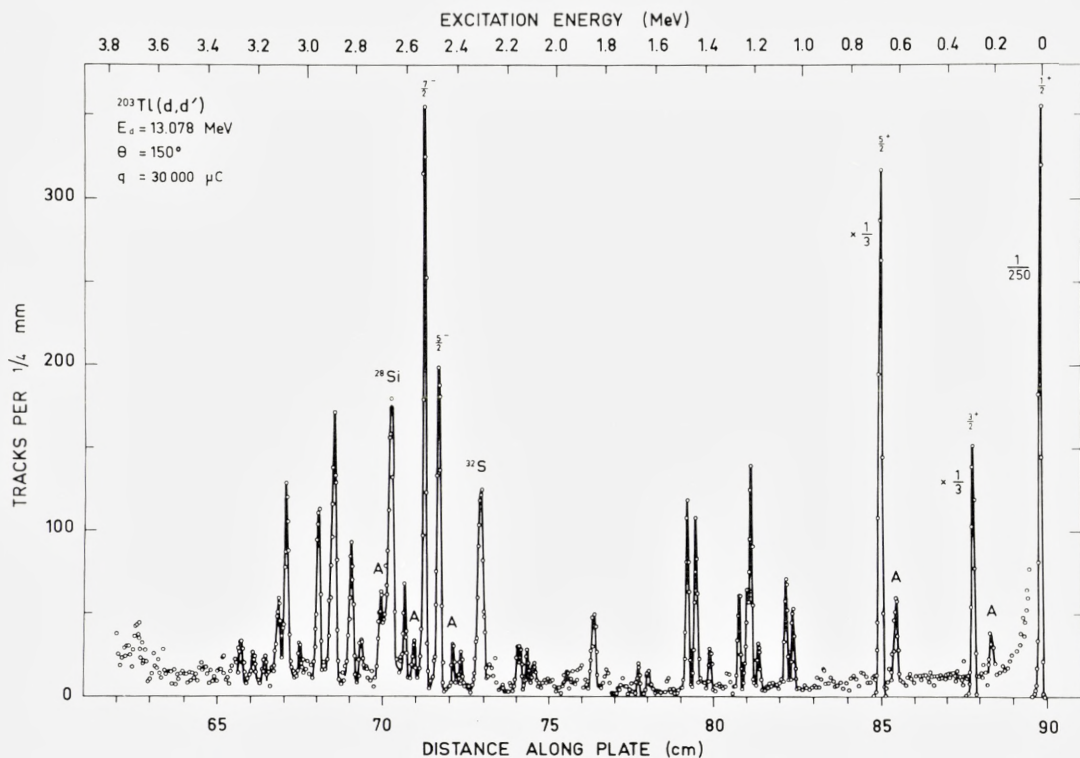


Fig. 10. Spectrum of deuterons scattered from ^{203}Tl . Peaks marked by the letter A are due to ^{205}Tl .

of the 120° to 150° cross sections indicates that the transitions to the stronger of these states are of multipolarity greater than two, but no definite identification is possible. Excluding the two strongest states at 2.483 and 2.539 MeV in ^{205}Tl and the octupole state at 2.618 MeV, the total inelastic cross section

TABLE 11. Relative Cross Sections of 5- Multiplet in ^{209}Bi

Energy (MeV)	Assignment A		Assignment B		Exp. (%)
	J	$2J+1$ (%)	J	$2J+1$ (%)	
2.988	13/2	12.7	13/2	12.7	12.5 ± 1.0
3.041	3/2	3.6	3/2	3.6	4.1 ± 0.5
3.091	5/2	5.5	5/2	5.5	5.4 ± 0.5
3.136	11/2 + 19/2	29.1	11/2 + 19/2	29.1	30.8 ± 1.2
3.154	17/2 + 7/2	23.7	17/2 + 9/2	25.5	23.6 ± 1.1
3.170	15/2	14.5	15/2	14.5	14.5 ± 1.0
3.213	9/2	9.1	1/2 + 7/2	9.1	9.2 ± 0.8

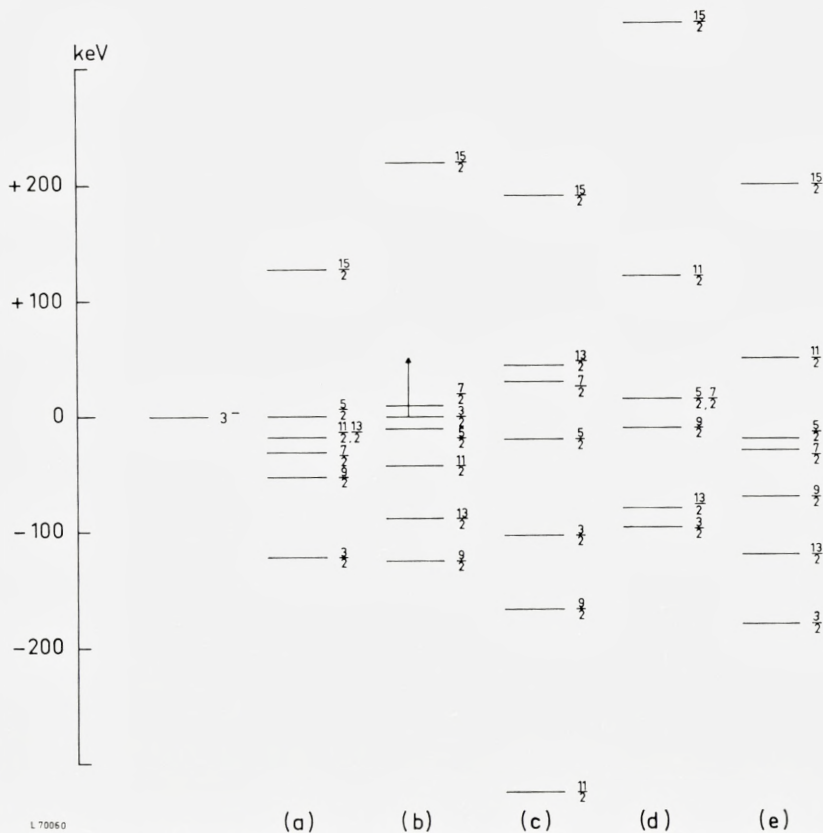


Fig. 11. Comparison of the experimental and calculated energy spectrum of the octupole septuplet in ^{209}Bi . Spectrum a) is the experimental spectrum while b) and c) are the results of particle-vibrational model calculations of HAMAMOTO¹⁵⁾ and BROGLIA et al.¹⁶⁾, respectively. Spectra d) and e) are the results of a shell-model calculation by ARITA and HORIE²⁸⁾.

in this energy region is about three times larger in ^{203}Tl than in ^{204}Pb . Including the octupole states, the cross sections are the same to within 25%. The most strongly populated peak above 2.6 MeV (at 2.899 MeV) is appreciably broader than adjacent groups and is probably a doublet.

A comment should be made about the states around 1.5 MeV of excitation. HINDS et al.³⁴⁾ observe a level at 1.48 MeV, which they suggest may be the major component of the $h_{11/2}^{-1}$ shell-model configuration. In the present spectra, a level is observed at 1.48 MeV which may correspond to this state. It may be noted, however, that the corresponding state at 1.479 MeV in ^{207}Tl is populated with a cross section that is a factor of four smaller.

The states at 1.446 and 1.481 MeV have a ratio of 120° to 150° cross sections, which is essentially the same as that observed for the octupole states

near 2.5 MeV, and it is interesting to consider the existence of a component in the ground-state wave function of ^{203}Tl which corresponds to a proton coupled to the ^{202}Hg core. Recent inelastic deuteron scattering studies⁴¹⁾ report the existence of a low-lying collective state in the even Hg isotopes, and the doublet of states formed by considering the coupling of the $s_{1/2}$ proton to this state could then be populated via this component of the ground state. However, it must be noted that the structure of this low-lying state in ^{202}Hg is unknown and, since such a state is not observed in the Pb isotopes studied, it would appear that the two proton holes acquired in a change from Pb to Hg play a major role in its wave function. It is not immediately clear that the addition of a proton to the Hg core would not then block the major component of the core vibration.

4. Conclusions

The results of the present study are schematically summarized in Fig. 12 which shows the inelastic cross sections observed for the levels below 3.5 MeV of excitation. The increase in number of levels populated with decreasing mass number is immediately evident. Also notable is the decrease in size of the strong groups near 2.6 MeV of excitation. The total inelastic cross sections observed below 3.5 MeV of excitation in each nuclide at a scattering angle of 150° are listed in table 12. It is interesting to note that this total does not vary greatly over the mass region studied. Of more particular interest is the comparison of the cross sections in the odd-mass isotopes to that in the even-mass cores. For ^{205}Tl and ^{203}Tl , the total is identical to that observed in the respective ^{206}Pb and ^{204}Pb cores considered in the coupling schemes. Considering ^{209}Bi , ^{208}Pb , and ^{207}Pb , where the weak-coupling description appears to be more valid, the agreement is not as marked, but the sums still do agree within 20 %.

The states corresponding to the highly collective octupole vibration at 2.615 MeV in ^{208}Pb dominate the high energy portion of each spectrum. Except for ^{203}Tl , these states have been previously identified. The small energy spread of these states and the large cross sections observed for them make possible an accurate comparison of the relative population strengths. These results, corrected for the small Q -value dependence and normalized to ^{208}Pb , are summarized in Fig. 13 and Table 13. Also listed in the table are the results of several other inelastic scattering studies and the octupole strength calculations of HAMAMOTO³⁰⁾. The present results show a larger decrease in strength away from ^{208}Pb than observed by other methods, but the large experimental

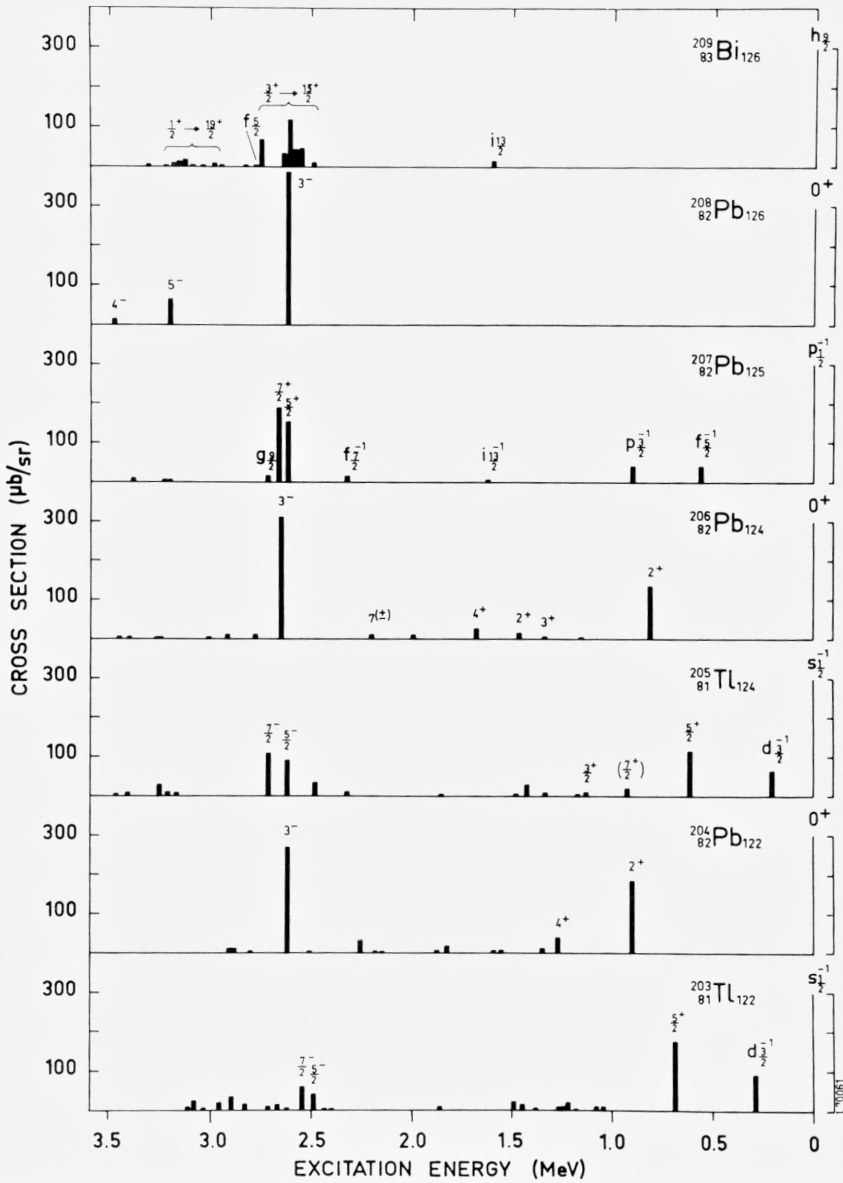


Fig. 12. Comparison of the level spectra below 3.5 MeV excitation populated at 120° in the present study.

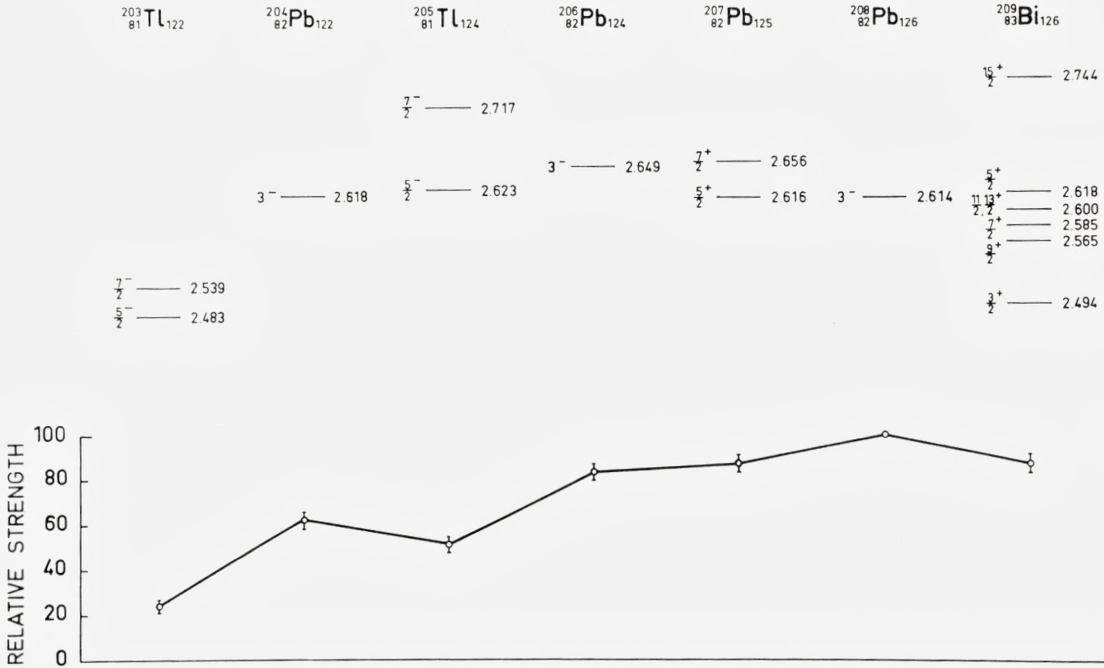


Fig. 13. Positions and relative strengths of the octupole states corresponding to the 2.614 MeV state in ^{208}Pb . The ^{208}Pb strength is normalized to 100.

errors which exist in earlier data do not yet allow any definite conclusions as to whether a genuine difference exists. As previously indicated, the cross sections observed with both ^{203}Tl and ^{205}Tl are substantially smaller than for the Pb and Hg isotopes⁴¹⁾ of corresponding neutron number. This may indicate that some of the octupole strength expected in the doublets is transferred to other levels, but no conclusion is possible before the structure of these additional levels is further investigated.

The $2J + 1$ intensity rule has been used to assign the level spins of several of the particle-vibrational multiplets observed in the ^{208}Pb region. This rule which is obtained on the assumption of a very weak particle-vibrational coupling can be used as an indicator of the amount of mixing between multiplet

TABLE 12. Total 150° Inelastic Cross Section Below 3.5 MeV of Excitation

Nuclide	^{209}Bi	^{208}Pb	^{207}Pb	^{206}Pb	^{205}Tl	^{204}Pb	^{203}Tl
Cross section <i>μb/sr</i>	430	490	580	570	560	620	620

TABLE 13. Relative Strengths Observed in Octupole Multiplets

	²⁰³ Tl	²⁰⁴ Pb	²⁰⁵ Tl	²⁰⁶ Pb	²⁰⁷ Pb	²⁰⁸ Pb	²⁰⁹ Bi
(<i>d, d'</i>) 13 MeV (α, α') ^b 42 MeV (<i>e, e'</i>) ^c 28–73 MeV (<i>p, p'</i>) ^d 24.5 MeV Predicted ^e)	24.8 ± 2.5	62.1 ± 3.0 87 ± 13 87	51.5 ± 3.0	83.4 ± 3.5 100 ± 12 96	86.8 ± 3.5 100 ± 10 89 ± 6 93	100 100 ± 6 93 ± 6 91	86.9 ± 3.5 89.7 ± 4.0 ^{a)} 100 ± 10 93 ± 7 100

a) Including $i_{13/2}$ population at 1.608 MeV.

b) Ref. 2). c) Ref. 10). d) Ref. 3). e) Ref. 30).

TABLE 14. Adherence to $2J + 1$ Cross Section Rule for Octupole Multiplets

Nuclide	Energy (MeV)	<i>J</i>	$2J+1$ (%)	Experiment (%)	Ratio of Experiment to $2J + 1$		Predicted ratio ^{b)}
					Present study	(<i>p, p'</i>) ^{a)}	
²⁰⁹ Bi	2.494	3/2	5.7	4.2 ± 0.3	0.74 ± 0.06	0.72	0.93
	2.566	9/2	14.3	13.8 ± 0.6	0.97 ± 0.05	1.06	1.07
	2.585	7/2	11.4	12.3 ± 0.5	1.08 ± 0.04	1.11	0.99
	2.600	11/2 + 13/2	37.2	37.4 ± 0.0	1.00 ± 0.02	1.01	1.04
	2.618	5/2	8.6	9.1 ± 0.5	1.06 ± 0.06	1.01	1.00
	2.744	15/2	22.8	23.7 ± 0.7	1.02 ± 0.04	0.95	0.97
²⁰⁷ Pb	2.616	7/2	57.1	58.3 ± 0.6	1.02 ± 0.02	1.00	1.00
	2.565	5/2	42.9	41.7 ± 0.6	0.97 ± 0.02	1.00	1.00
²⁰⁵ Tl	2.717	7/2	57.1	57.7 ± 0.8	1.01 ± 0.02		
	2.623	5/2	42.9	42.3 ± 0.8	0.99 ± 0.02		
²⁰³ Tl	2.539	7/2	57.1	62.4 ± 1.0	1.09 ± 0.02		
	2.483	5/2	42.9	37.6 ± 1.0	0.88 ± 0.03		

a) Ref. 3). b) Ref. 15).

members and other configurations. Of the multiplets observed, only those corresponding to the octupole states near 2.6 MeV have been identified in all four odd-mass nuclei studied and the adherence to this rule is summarized in Table 14. Except for the weak $3/2+$ state in ²⁰⁹Bi, the agreement with the rule is quite good. As previously noted, the admixtures into the doublet in ²⁰³Tl appear to be somewhat larger than in ²⁰⁹Bi, ²⁰⁷Pb, and ²⁰⁵Tl.

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